CFAR STAP Radar using Compressive Sensing for Random Arrays

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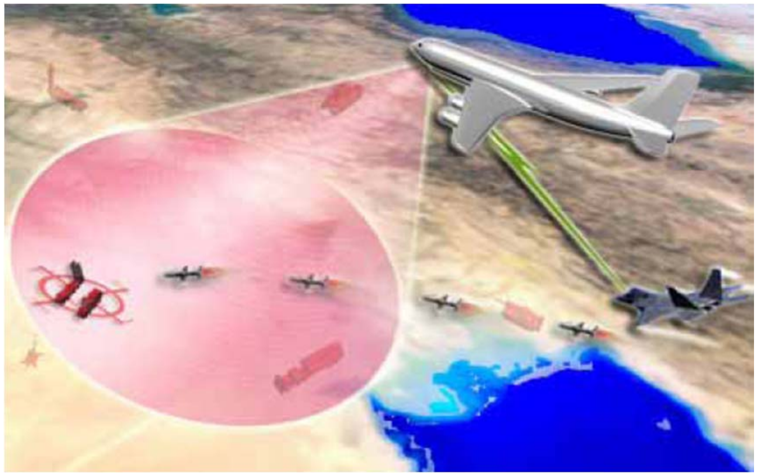
Outline

- Signal model
- Sparse random arrays
- Sparsity-based CFAR detectors
- Numerical Results
- Conclusions

Introduction to GMTI

- Goal: Detect targets on the ground in the presence of ground clutter.
- Movement of targets helps distinguish targets from ground clutter.
- Challenge: Slow moving targets are difficult to detect in ground clutter.

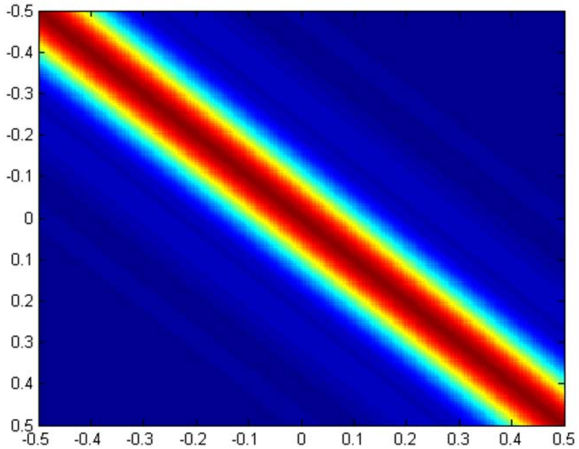


From Northrop-Grumman presentation on GMTI

Main idea

- Doppler visibility of targets improves with increasing array aperture.
- Increase array aperture but not the number of array elements.
  - Undersampled array ➡ spatial compressive sensing
  - Side effects: grating lobes or high random sidelobes
- Proposed solution: exploit sparsity to

  prevent false alarms due to sidelobes.
- Develop a CFAR procedure



Sparse framework

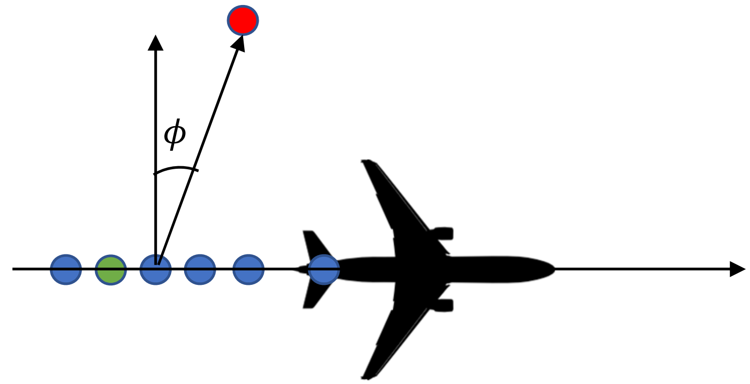
Consider an array with:

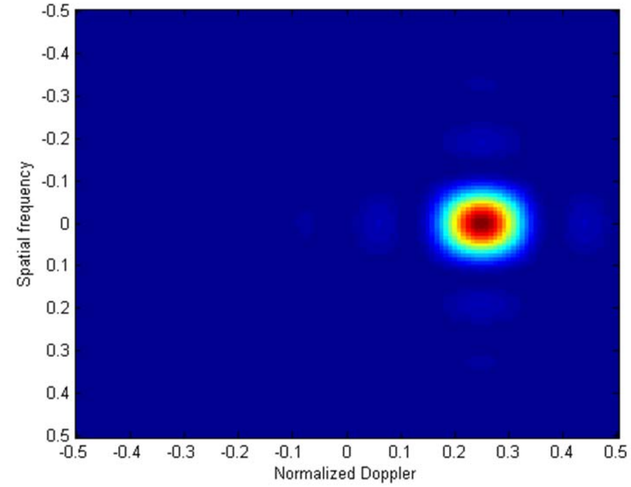
- N sensors
- A transmitter waveform with P pulses with CPI T_r

- $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e}$

- $\mathbf{A}$ - $NP \times G$ matrix of steering vectors
- $\mathbf{x}$ - $G \times 1$ vector with $K \ll G$ nonzeros
- Underdetermined $NP \ll G$
- $\mathbf{e}$ – Colored Gaussian noise - $CN\sim(\mathbf{0}, \mathbf{R})$

- Beampattern $P(u, f) = |\mathbf{a}(u, f)^H \mathbf{y}|^2$





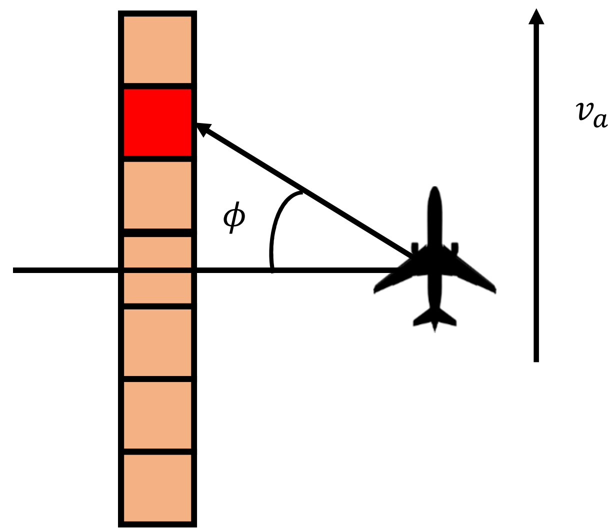
Example target response

Clutter interference

- Platform velocity - v_a
- For a clutter patch at spatial frequency $u = \sin\phi$
- Doppler shift of clutter patch:

$$f(u) = \frac{2v_a T_r}{\lambda} u$$

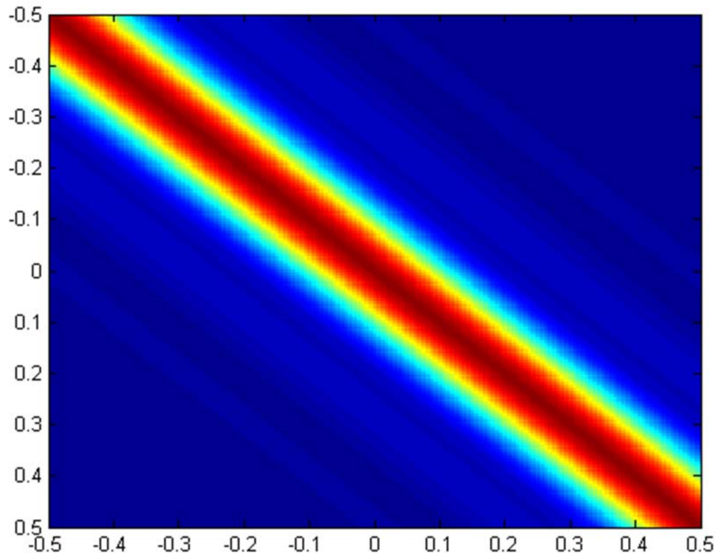
- Clutter patch at spatial frequency u has a Doppler shift $f(u)$ induced by the motion of the aircraft.



Clutter ridge

- Clutter response:

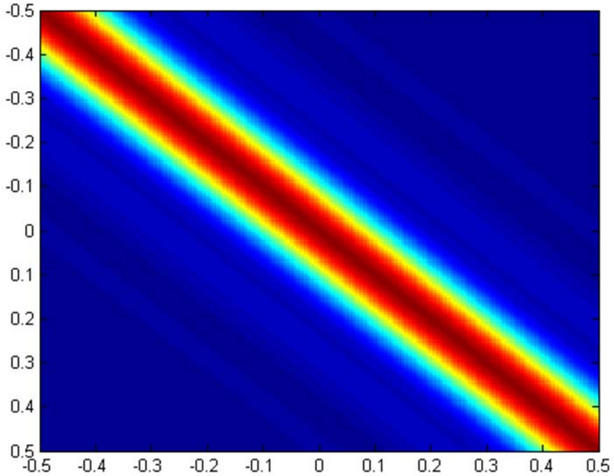
$$\mathbf{x}_c = \sum_{i=1}^{N_c} \alpha_i \mathbf{a}(u_i, f_i)$$



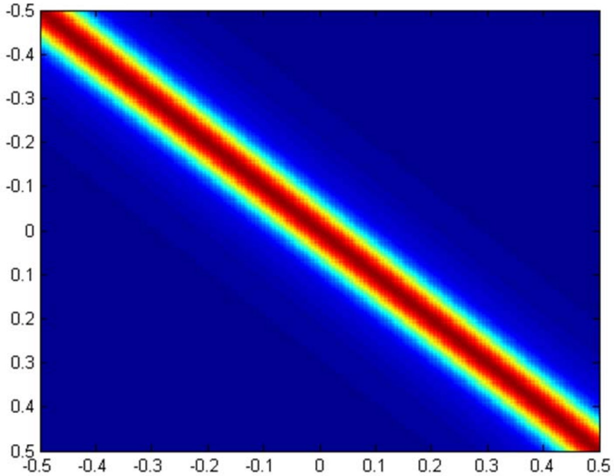
- Clutter response forms a diagonal ridge referred to as clutter ridge.

- Challenge: Slow moving targets near the clutter may be masked by clutter.

Array considerations



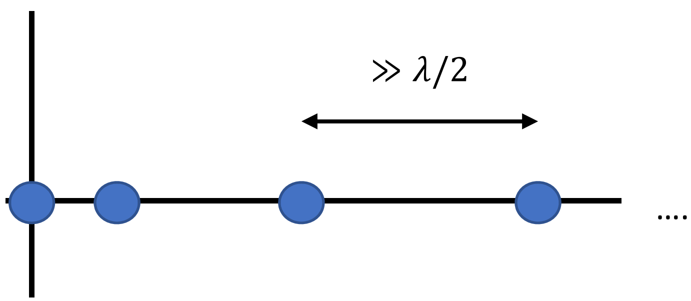
5λ ULA

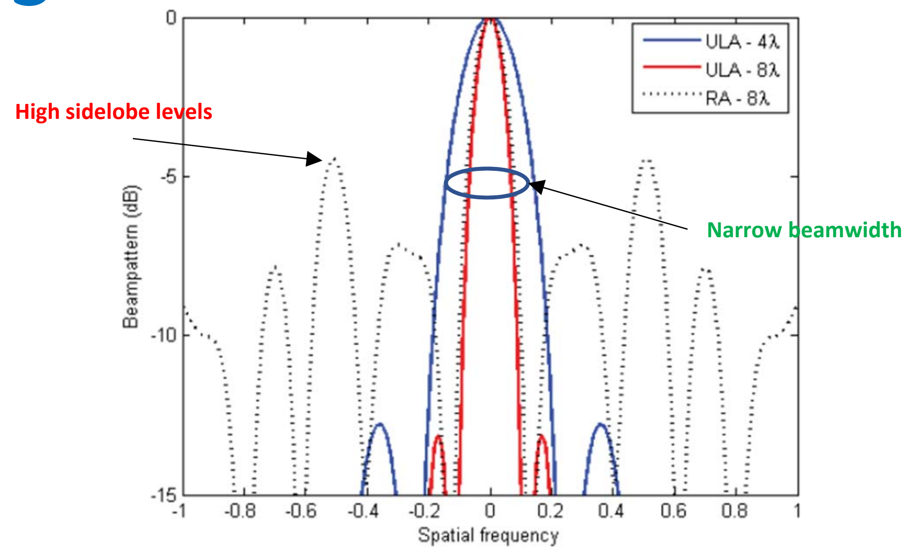


15λ ULA

- The width of the clutter ridge depends on the array aperture.
- Problem: Number of sensors increases *linearly* with the aperture of the array.

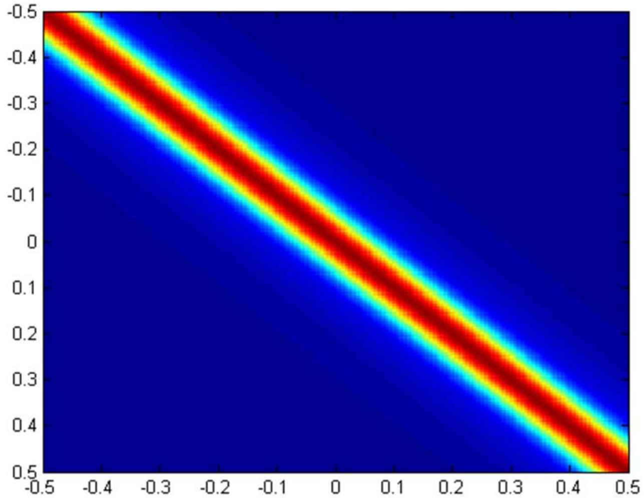
Sparse random arrays



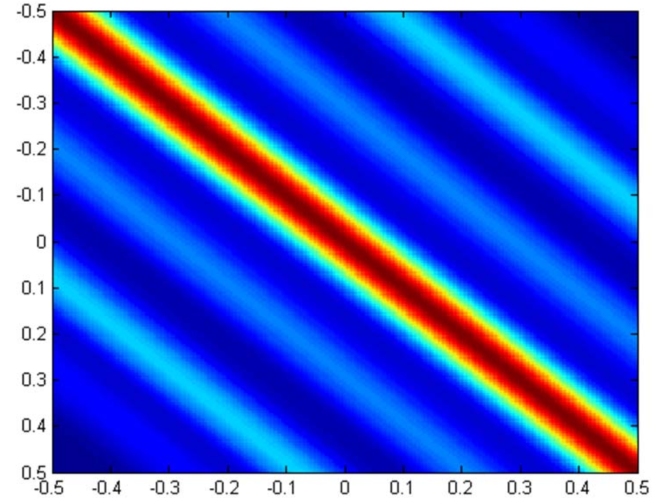


- Sparse random arrays distribute a small number of receivers randomly across a large array.
- Ergodicity – statistics of array response may be gleaned from single array realization
- Pros: Allows one to obtain a high resolution radar with fewer elements.
- Problem: Undersampling generates high sidelobes.

Clutter ridge with sparse random arrays



15λ ULA

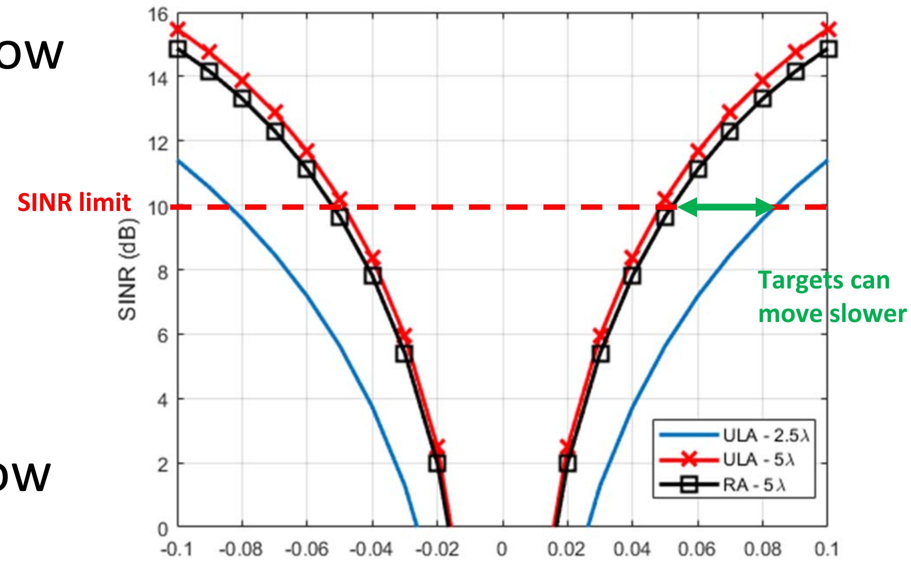


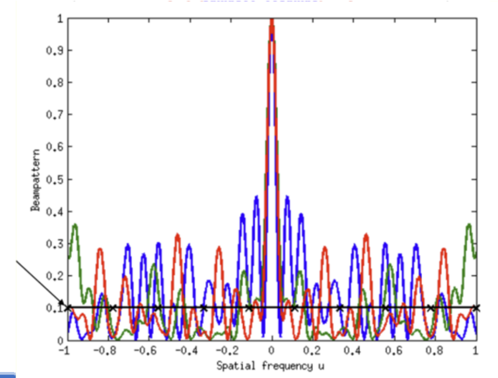
15λ Sparse random array

Clutter ridge shrinks due to increased aperture, but sidelobes of the clutter appear.

Minimum detectable velocity

- SINR vs Doppler: A measure of how slow target can be moving while still being detected.
- Clutter patch at zero Doppler.
- Small ULA has the widest response, slow targets present reduced SINR
- Sparse random array has similar mainlobe response to large ULA ⟹ similar detection performance of slow targets.





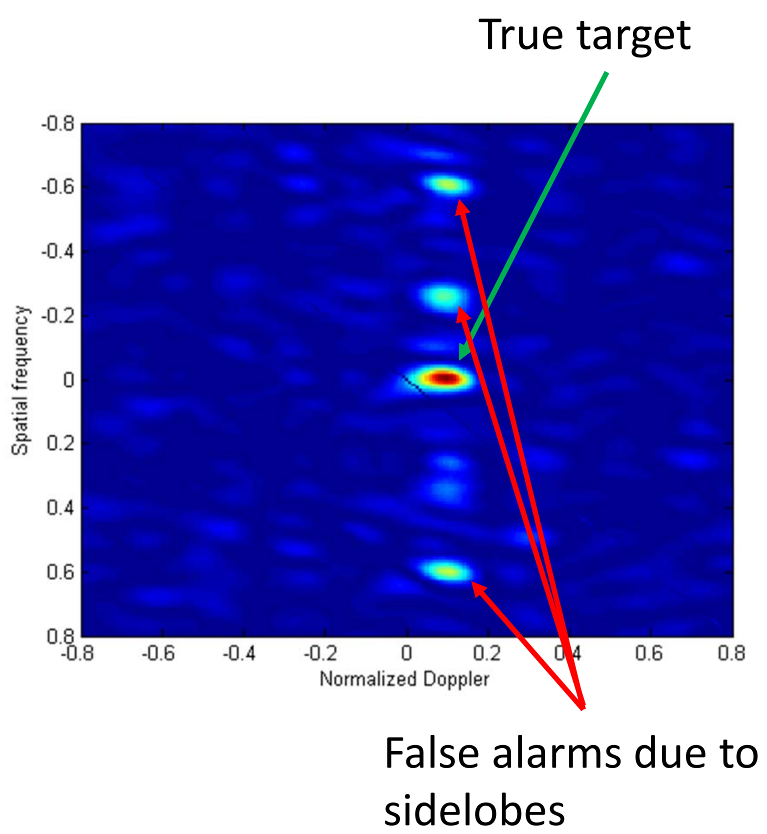
CFAR STAP with random arrays

- CFAR STAP:

$$T = \frac{\left|\mathbf{a}(u,f)^H \widehat{\mathbf{R}}^{-1} \mathbf{y}\right|^2}{\mathbf{a}(u,f)^H \widehat{\mathbf{R}}^{-1} \mathbf{a}(u,f)} \geq \gamma$$

- Pros: High resolution improves ability to detect slow moving targets.

- Problem: The high sidelobes of the sparse random array may cause high false alarm rates.



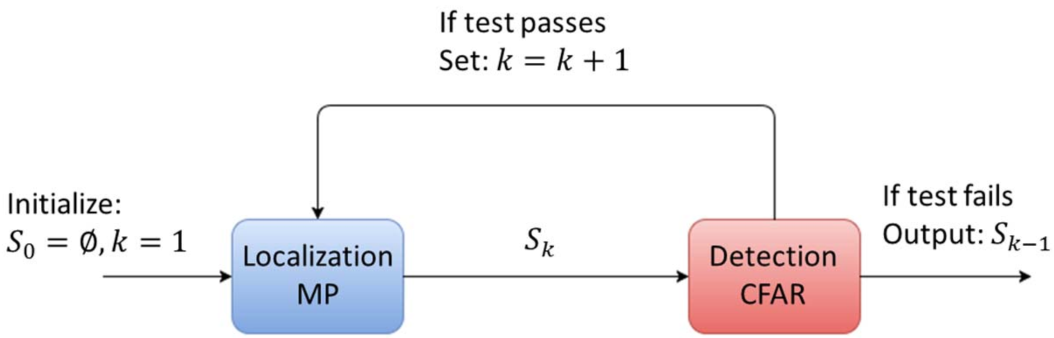
Sparsity based detectors

- Explain undersampled data assuming a small number of targets
- Perform whitening:
  - Whitened dictionary $\mathbf{B} = \widehat{\mathbf{R}}^{-1/2}\mathbf{A}$
  - Whitened observations $\mathbf{z} = \widehat{\mathbf{R}}^{-1/2}\mathbf{y}$
- Optimization problem:

$$\min_{\mathbf{x}} \|\boldsymbol{z} - \mathbf{B}\mathbf{x}\|_2^2 \ \text{subject to} \ \|\mathbf{x}\|_0 \leq K$$

- Problem: Constraint is non-convex and requires information on the number of targets which is unknown.

MP-CFAR

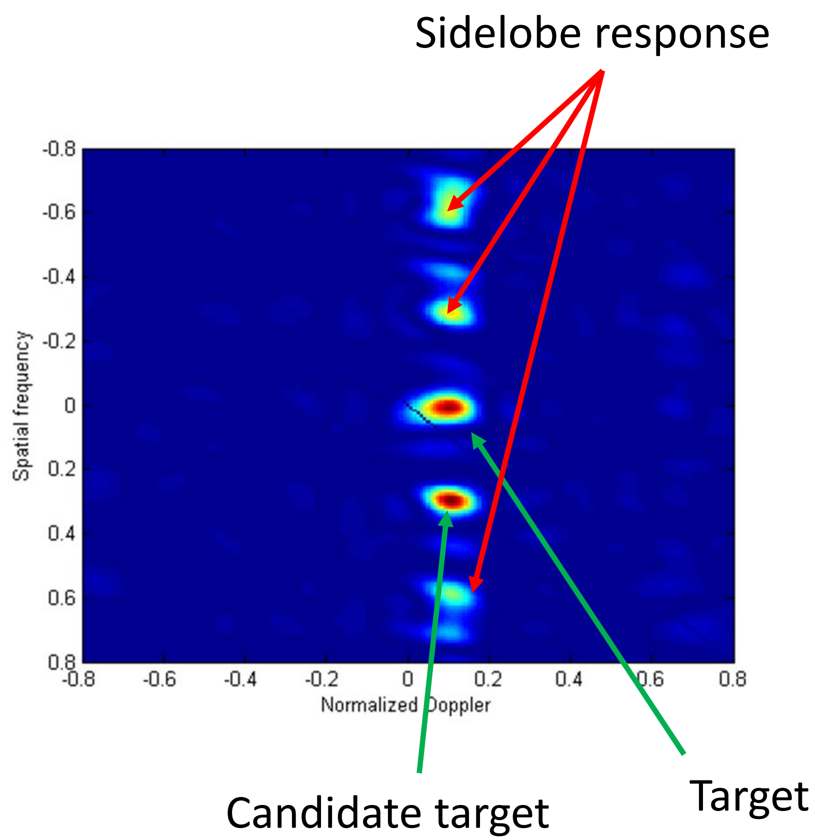


- Matching pursuit combined with a detection test
  - First stage: Perform sparse localization using MP.
  - Second stage: Apply CFAR test to target obtained in first stage
- Effect of detected targets removed from the data.
- MP-CFAR terminates when a target fails detection test.

MP localization

- Idea: Detect strongest target first:

$$m_1 = \max_j \frac{|\mathbf{b}_j^H \mathbf{z}|^2}{\left\|\mathbf{b}_j\right\|_2^2}$$



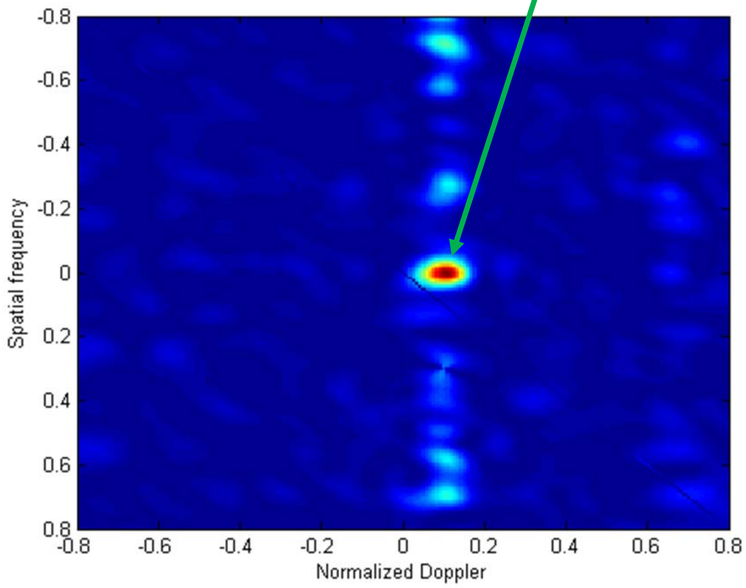
MP-localization

- Once detected, target effect is removed to prevent rediscovery through sidelobes.
- Next strongest target is detected:

$$m_2 = \max_j \frac{|\mathbf{w}_j^H \mathbf{z}|^2}{\|\mathbf{w}_j\|_2^2}, \quad j \notin m_1$$

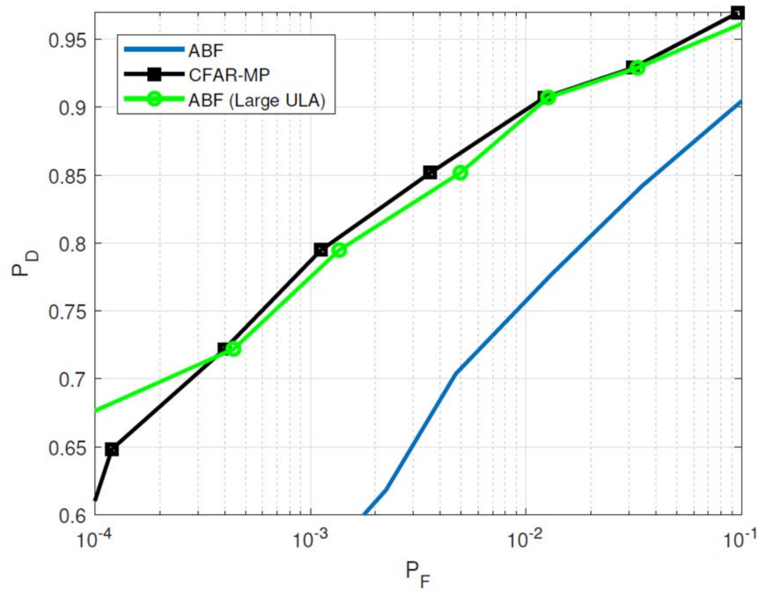
- $\mathbf{w}_j = \mathbf{P}^{\perp}_{\mathbf{B}_{S_{k-1}}} \mathbf{b}_j$
- $\mathbf{P}^{\perp}_{\mathbf{B}_{S_{k-1}}} = \mathbf{I} - \mathbf{B}_{S_{k-1}} \left(\mathbf{B}^H_{S_{k-1}} \mathbf{B}_{S_{k-1}}\right)^{-1} \mathbf{B}^H_{S_{k-1}}$

Next candidate target



ROC for a single slow moving target

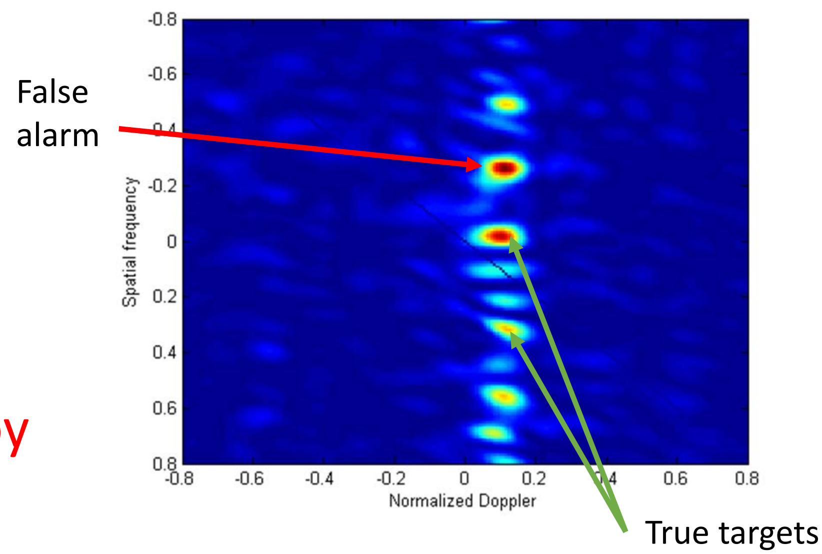
- $N = 12, P = 16, K = 1, Z = 8\lambda$
- ABF with sparse array performs the worst, due to large sidelobes.
- ABF with large ULA performs well as it was intended.
- MP-CFAR performs similarly to ABF with large ULA.



Conclusions

- Sparse random arrays support high resolution radar without the need for a large number of sensors, but at the cost of increased sidelobes.
- MP-CFAR uses MP to obtain a sparse localization solution then applies the proposed CFAR detector to perform detection.
- MBMP-CFAR generalizes MP-CFAR and allows one to obtain higher performance at the cost of increased computational complexity.
- Both sparsity algorithms support CFAR detection of targets with a sparse array.

MBMP-CFAR

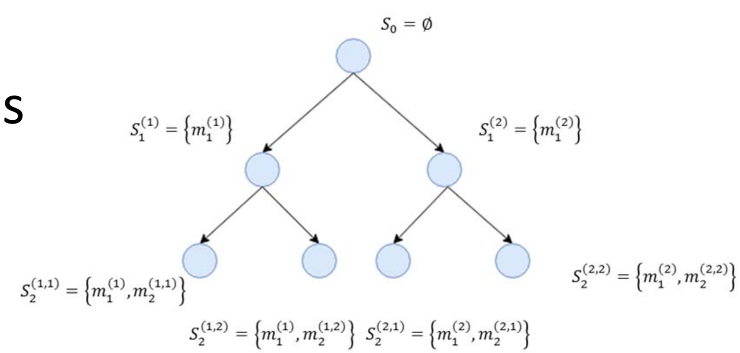


- MP only tests targets with the largest projection.

- If an incorrect resolution cell is tested by MP-CFAR, the probability of false alarm increases.

- MBMP-CFAR generalizes MP-CFAR and tries more combinations.

- Probability that MBMP-CFAR obtains the correct set of targets is higher.



Probability of false alarm

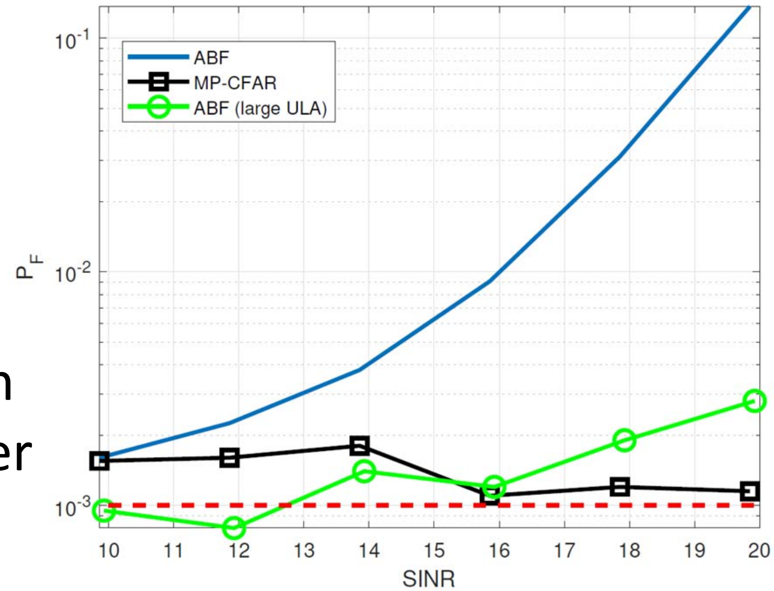
- $N = 12, P = 16, K = 1$

- Thresholds are set so that each method achieves $P_F = 10^{-3}$.

- ABF using sparse random array suffers from high sidelobes and experiences much higher P_F.

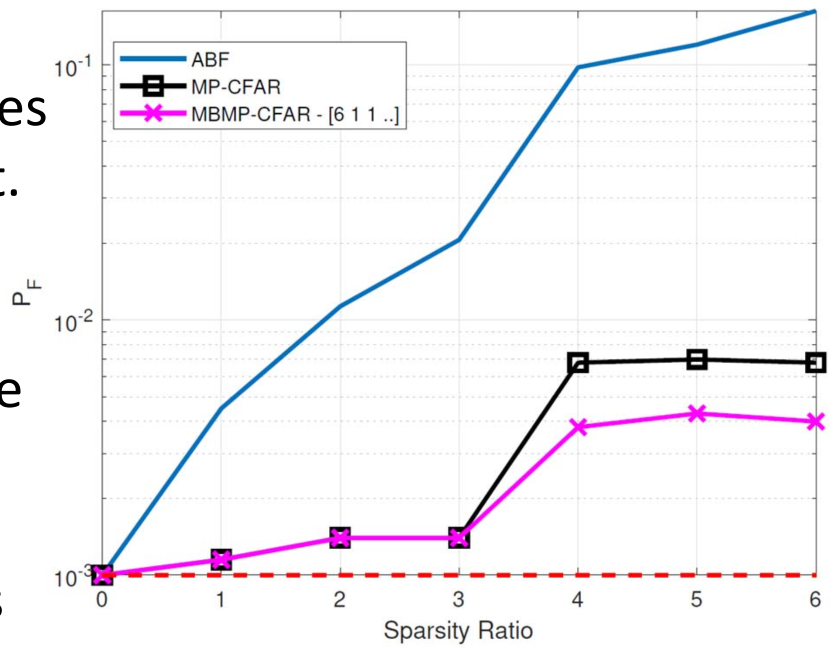
- ABF with a large ULA works as intended, slight increase in P_F at higher SNRs.

- MP-CFAR performs at desired P_F even with high sidelobes.



CFAR for multiple targets

- $N = 12, P = 16, K = 2, Z = 8\lambda$
- The false alarm rate of the ABF increases significantly when any target is present.
- Both MP-CFAR and MBMP-CFAR can detect about $K = 3$ targets before false alarm rate is higher than expected.
- MBMP-CFAR can detect up to 6 targets experiencing about $P_F \geq 3 \times 10^{-3}$.



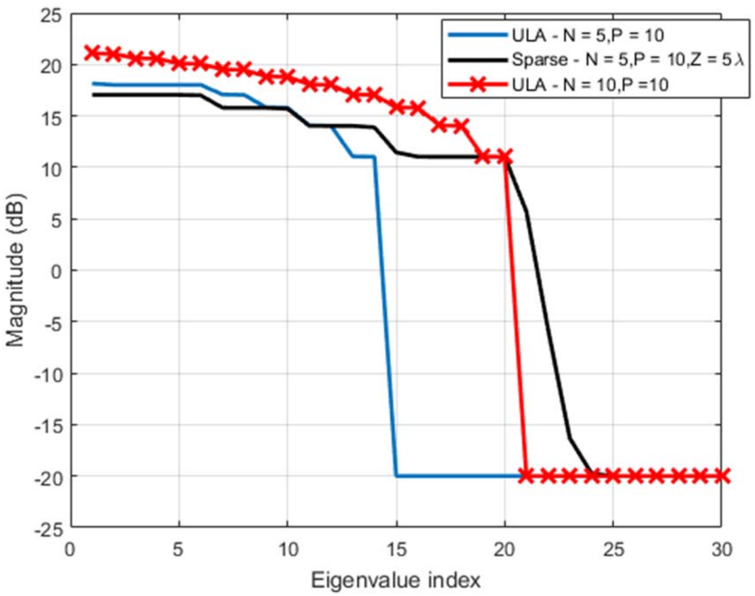
Questions?

Clutter rank

- Covariance matrix of interference:

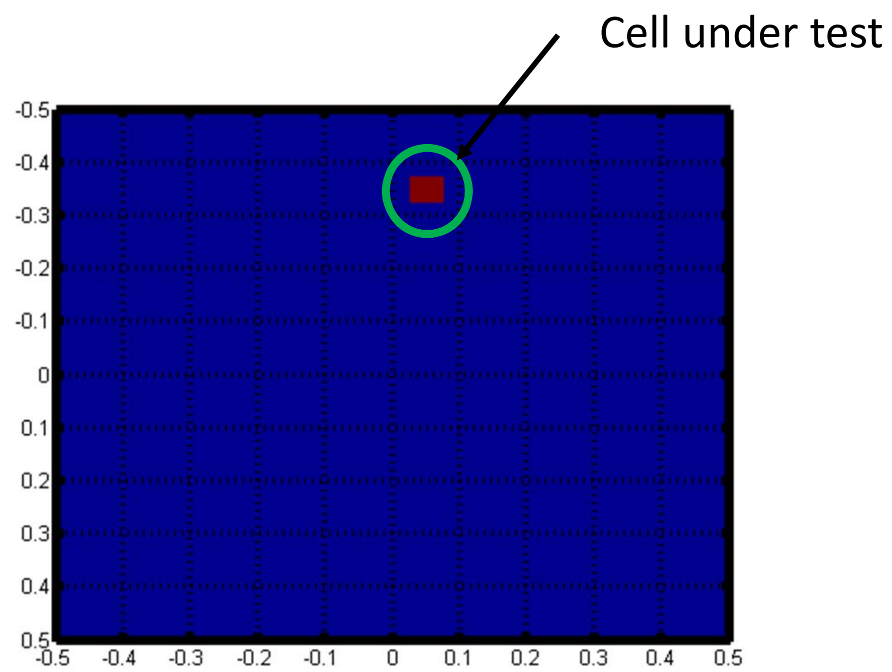
$$\mathbf{R} = \mathbf{R}_c + \mathbf{R}_n$$

- Rank of $\mathbf{R}_c$ is an indicator of how many DOF is required to suppress clutter.
- Sparse random array requires as many DOFs as a large ULA.
- Sparse random array has fewer DOFs available and provides less gain.



Detection with sparse random arrays

- Divide the angle-Doppler map into G cells and perform detection test for all cells.
- $T = \frac{\left|\mathbf{a}(u,f)^H \mathbf{R}^{-1} \mathbf{y}\right|^2}{\mathbf{a}(u,f)^H \mathbf{R}^{-1} \mathbf{a}(u,f)} \geq \gamma$
- All resolution cells that exceeds the threshold are declared as targets.
- Test requires knowledge of interference covariance matrix.



Interference covariance matrix

- In practice, the interference covariance matrix is not available and must be estimated.
- Secondary data that is known to be target-free is used to estimate interference covariance matrix.
- Estimate interference covariance matrix:

$$\widehat{\mathbf{R}} = \frac{1}{L}\sum_{l=1}^{L} \mathbf{q}_l \mathbf{q}_l^H, \quad L > N$$

- Replacing $\mathbf{R}$ with $\widehat{\mathbf{R}}$ yields the adaptive beamformer.

CFAR detector

- Hypothesis test for CFAR detector:

$$H_0: \quad x_i = 0$$
$$H_1: \quad x_i \neq 0$$

- Test is applied to all targets in S_k.

- GLRT: $T = \dfrac{\max_{\mathbf{x}_{S_k}} p(\mathbf{y}|H_1)}{\max_{\mathbf{x}_{S_k \setminus i}} p(\mathbf{y}|H_0)} \geq \gamma$

  $p(\mathbf{y}|H_1) = \dfrac{1}{\|\mathbf{R}\|\pi^N} e^{\left(\mathbf{y}-\mathbf{A}_{S_k}\mathbf{x}_{S_k}\right)^H \mathbf{R}^{-1}(\mathbf{y}-\mathbf{A}_{S_k}\mathbf{x}_{S_k})}$

- $T = \dfrac{\left|\mathbf{f}_i^H \mathbf{z}\right|^2}{\mathbf{f}_i^H \mathbf{f}_i} \geq \gamma, \quad \mathbf{f}_i = \mathbf{P}_{\mathbf{B}_{S_k \setminus i}}^{\perp} \mathbf{b}_i$

Handling clutter contributions

- Clutter contributions are much stronger than target contributions and may cause errors with localization.

- To remove clutter contributions, whiten data before solving CS problem.

- Perform whitening:

$$\mathbf{B} = \widehat{\mathbf{R}}^{-1/2}\mathbf{A}, \qquad \mathbf{z} = \widehat{\mathbf{R}}^{-1/2}\mathbf{y}$$

- Optimization problem: $\min_{\mathbf{x}} \|\boldsymbol{z} - \mathbf{B}\mathbf{x}\|_2^2$ subject to $\|\mathbf{x}\|_0 \leq K$

Comparison with ABF

- ABF test:

$$T = \frac{\left|\mathbf{a}(u,f)^H \widehat{\mathbf{R}}^{-1} \mathbf{y}\right|^2}{\mathbf{a}(u,f)^H \widehat{\mathbf{R}}^{-1} \mathbf{a}(u,f)} \geq \gamma$$

$$T = \frac{\left|\mathbf{b}(u,f)^{\mathrm{H}} \mathbf{z}\right|^2}{\mathbf{b}(u,f)^H \mathbf{b}(u,f)} \geq \gamma$$

- Proposed CFAR test:

$$T = \frac{\left|\mathbf{f}_i^H \mathbf{z}\right|^2}{\mathbf{f}_i^H \mathbf{f}_i} \geq \gamma; \quad \mathbf{f}_i = \mathbf{P}^{\perp}_{\mathbf{B}_{S_k \setminus i}} \mathbf{b}_i$$

$$T = \frac{\left|\mathbf{b}_i^H \mathbf{P}^{\perp}_{\mathbf{B}_{S_k \setminus i}} \mathbf{P}^{\perp}_{\mathbf{B}_{S_k \setminus i}} \mathbf{z}\right|^2}{\mathbf{b}_i^H \mathbf{P}^{\perp}_{\mathbf{B}_{S_k \setminus i}} \mathbf{P}^{\perp}_{\mathbf{B}_{S_k \setminus i}} \mathbf{b}_i} = \frac{\left|\mathbf{f}_i^H \overline{\mathbf{z}}\right|^2}{\mathbf{f}_i^H \mathbf{f}_i} \geq \gamma$$

$$\overline{\mathbf{z}} = \mathbf{P}^{\perp}_{\mathbf{B}_{S_k \setminus i}} \mathbf{z}$$

- Difference: Proposed CFAR test projects measurements away from previously detected targets.

Statistics of detectors

- ABF: $T = \frac{|\mathbf{b}^H \mathbf{z}|^2}{\mathbf{b}^H \mathbf{b}} \geq \gamma$ Proposed test: $T = \frac{|\mathbf{f}_i^H \bar{\mathbf{z}}|^2}{\mathbf{f}_i^H \mathbf{f}_i} \geq \gamma$
- Probability of false alarm for ABF:

$$P_F = 1 - \int_0^1 F_{2,2(L-N+1)}(\gamma)\, p(h) dh$$

- $F_{2,2(L-N+1)}(\gamma)$ – CDF of F distribution $p(h)$ – PDF of beta distribution $h - \beta(L+1, N-1)$

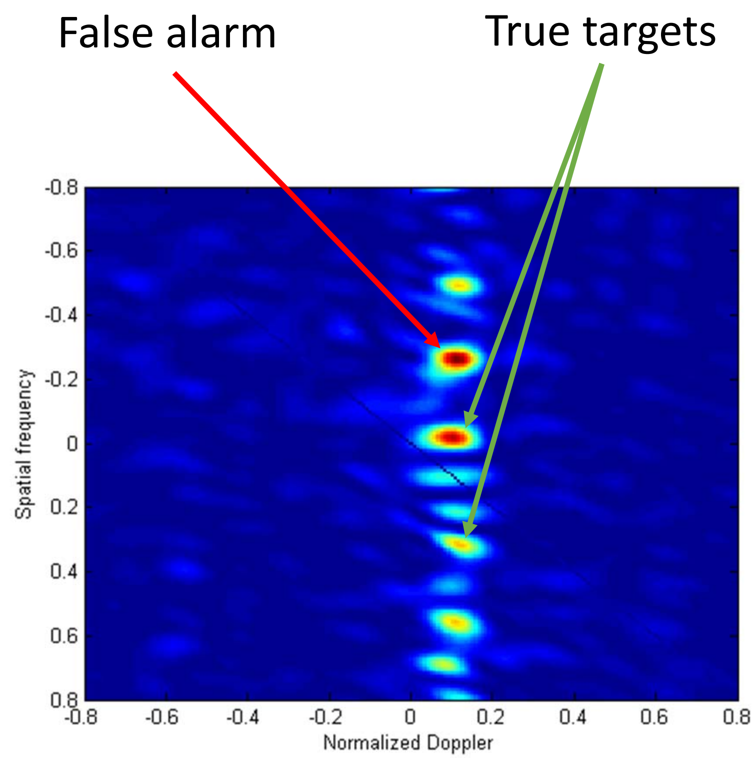
- Approximate the probability of false of proposed test the same as ABF:

$$P_F = 1 - \left(\int_0^1 F_{2,2(L-N+1)}(\gamma)\, p(h) dh \right)^G$$

- The power G captures effect of taking maximum due to MP.

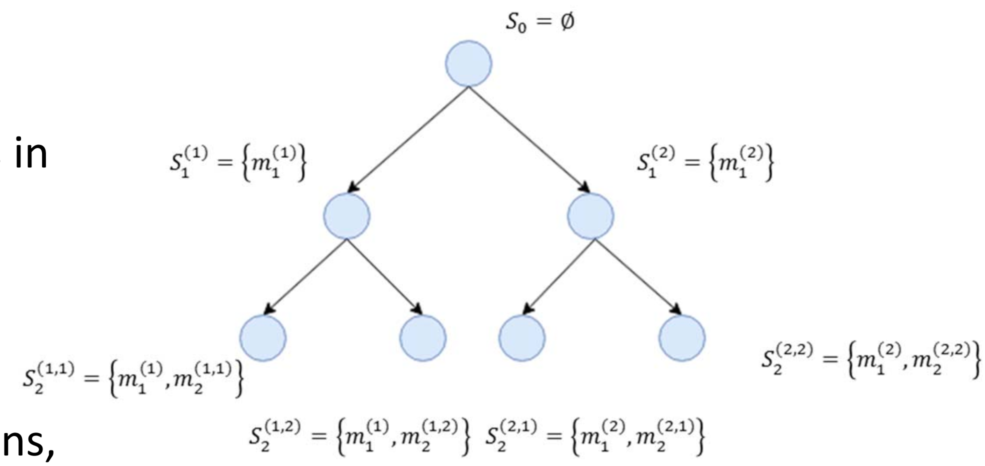
Errors with MP-CFAR

- MP only tests targets with the largest projection.
- Problem: If an incorrect resolution cell is tested by MP-CFAR, the probability of false alarm will increase.
- This error may also cause further errors in subsequent detections.



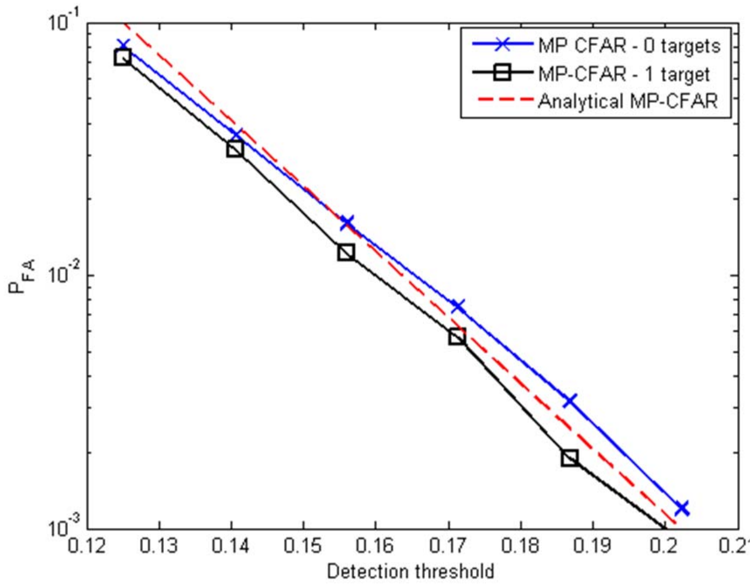
MBMP-CFAR

- Solution: Obtain multiple sparse solutions in stage 1 and tests the "best" candidate.
- MBMP-CFAR generalizes MP-CFAR.
- Since, MBMP-CFAR tries more combinations, the probability that MBMP-CFAR obtains the correct set of targets is higher.



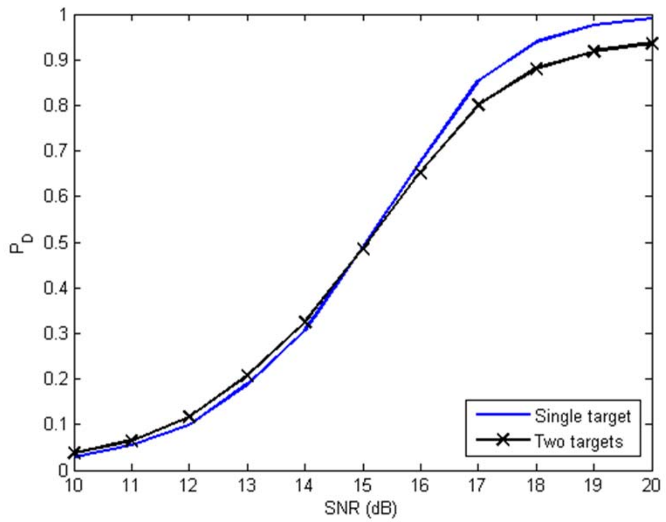
MP-CFAR false alarm probability

- Probability of false alarm is not significantly affected by the number of targets.
- A threshold can be designed to achieve a desired false alarm probability even when sidelobes are large.



MP-CFAR detection probability

- Detection performance does not change significantly as the number of targets change.
- Small performance loss is seen for two targets due to modifying steering vectors to remove previously detected targets.
- As the number of targets increases the detection performance will decrease.



ROC two slow moving targets

- $N = 12, P = 16, K = 2, Z = 8\lambda$
- ABF with small ULA and Sparse random array performs poorly as before.
- MP-CFAR performs similarly to ABF with large ULA for $P_F \geq 10^{-3}$.
- MBMP-CFAR experiences lower P_F compared to MP-CFAR